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EVALUATION OF DESIGN METHODS FOR GEOMETRIC CONTROL

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ABSTRACT

control

Geometric control can produce desirable control by decoupling the input disturbances from the selected output variables. The basic principle for this method was originally introduced by Wonham. The mathematical complexity involved, however, makes the method very hard to get accepted by the chemical community. The paper evaluates Wonham's original method together with three other methods, i.e. eigenvalue/eigenvector methods by Shah et al, the graph theory by Schizas and Evans and the simplified method by Kümmel et al. The evaluation considers the basic potential of the methods, the prerequisite of the designer, transparency, computer demand, and potential for pole shift.

INTRODUCTION

A new approach to multivariable control called "A Geometric Approach" has been developed by Wonham and his coworkers refs. (1, 2, and 3). In this approach it is recognized that the properties of a linear system depend on the structure of the linear subspace generated by the given system in the state space. The design problem of a control system is then regarded as synthesizing a system in such a way that the resulting system, which includes the control system, will generate a linear subspace which has the desirable structure in the state space so as to satisfy the design specification.

By describing the design specifications of the feedback controller as the structure of the feedback subspace generated by the controller system, the design is treated more intuitively, and better insight is obtained as to the conditions for the existence of a solution and other such problems as decoupling control and disturbance localization, refs. (4 and 5).

Even with these contributions, the application of geometric control is not widespread, possibly due to the abstract nature of the underlying theory. Assuming this is the case, there is a need for a simplified design method. Such a method has been developed by ref. (6) resulting in better insight into and simpler design of the geometric controller.

Geometric control has been implemented on some processes like evaporators refs. (5 and 7). In ref. (8) the theoretical results for disturbance rejection on a distillation column based on Wonham's results have been developed, and they have evaluated the results through simulation. Also in refs. (5 and 9) this problem has been considered. A graph theoretic approach to the same problem is developed in ref. (10).

In this paper the four above mentioned design methods are evaluated and compared in order to offer some guidance as to what method to prefer. The evaluation considers the basic potential of the methods, the prerequisite of the designer, transparency, computer demand and potential for poleshift.

THE GEOMETRIC APPROACH

The following development is a summary of the results in ref. (8) with a slightly different notation. The multivariable system is described by

$$\dot{\underline{x}}(t) = A\underline{x}(t) + B\underline{u}(t) + D\underline{q}(t); \qquad \underline{y}(t) = C\underline{x}(t), \tag{1}$$

where the state vector $\underline{x}(t)$ has dimension n, the control vector $\underline{u}(t)$ dimension m, the output vector $\underline{y}(t)$ dimension p, and the disturbance vector $\underline{q}(t)$ dimension d (where $p \leq m$). The matrices A(nxn), B(nxm), C(pxn), and D(nxd) are time independent. The problem is now to find the feedback matrix F(mxn) in

$$\underline{u} = F\,\underline{x} \tag{2}$$

so that any disturbance $\underline{q}$ has no influence on the controlled output $\underline{y}$.

Note that through this definition, C may be different from the usual output matrices. Insertion of eq. (2) into eq. (1) yields

$$\dot{\underline{x}} = (A+BF)\underline{x} + D\underline{q}\,, \tag{3a}$$

$$\underline{y} = C\,\underline{x}\,. \tag{3b}$$

With the initial condition $\underline{x} = \underline{0}$, the output vector $\underline{y}$ is obtained from eqs. (3a,b) as

$$y(t) = \sum_{k=0}^{n-1} C(A+BF)^k D\int_0^t \beta_k(t-\tau)\underline{q}(\tau)d \tag{4}$$

where $\beta_k(t)$ is a scalar function (which can be determined through Cayley-Hamilton's theorem). For the output variable $\underline{y}$ to be zero, one must require

$$C(A+BF)^k \mathbf{D} = \{\underline{0}\}, \quad k=0,1,2,\ldots,n-1 \tag{5}$$

where $\mathbf{D}$ is the range of matrix D (i.e. $\mathbf{D}$ is spanned by the columns of D). Eq. (5) is equivalent to

$$(A+BF)^k \mathbf{D} \subseteq \mathbf{N}(C), \quad k=0,1,2,\ldots,n-1 \tag{6}$$

where $\mathbf{N}(C)$, the null-space of C, is defined as

$$\mathbf{N}(C) = \{\underline{x} \in R^n : C\,\underline{x} = \underline{0}\}. \tag{7}$$

Eq. (6) is satisfied if

$$\mathbf{D} \subseteq \mathbf{N}(C) \tag{8a}$$

$$(A+BF)\,\mathbf{D} \subseteq \mathbf{D} \tag{8b}$$

and it can be shown that there exists a matrix F which satisfies (8b) if, and only if

$$\mathbf{D} \subseteq \mathbf{V}^*, \tag{9}$$

where $\mathbf{V}^*$ is the supremal (A,B)-invariant subspace in $\mathbf{N}(C)$ (see ref. (3)). V* can be obtained by the sequence:

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2) If p=m then it can be shown that Λ_2 consists of the eigenvalues of $A_{22}-B_2B_1^{-1}A_{12}$. This means that Λ_2 is fixed and only Λ_1 must be chosen

The remaining submatrices W_{21} and W_{22} can be computed from the equations:

$$W_{21}\,\Lambda_1 - P\,W_{21} = T\,W_{11} + S\,W_{11}\,\Lambda_1 \tag{25}$$

$$W_{22}\,\Lambda_2 - P\,W_{22} = T\,W_{12} + S\,W_{12}\,\Lambda_2 \tag{26}$$

where $S = B_2\,B_1^{-1}$, $T = A_{21} - S\,A_{11}$ and $P = A_{22} - S\,A_{12}$.

Next, the feedback matrix can be calculated from

$$F = (B^T\,B)^{-1}\,B^T(W\,\Lambda\,V - A) \tag{27}$$

The importance in the method of Shah et al. is that instead of determining the supremal (A,B)-invariant subspace through (10) and then solve (12), the closed-loop eigenvalues and eigenvectors are chosen, and (25-26) are solved.

Finally, the feedforward matrix K_{FF} is calculated from (19) and (22).

SIMPLIFIED GEOMETRIC DESIGN

The geometric design method as presented above may appear to be difficult due to the abstract nature of the underlying theory, especially the synthesis of the supremal (A,B)-invariant subspace $\mathcal{V}^*$. An alternative design procedure which is easier to handle and which also gives a better insight into what is required concerning the system outputs to obtain total disturbance rejection has been developped, see ref. (6).

Let again C have the form (20). For the analysis below, three submatrices A*, B* and D* are introduced.

Definition

The submatrices A*(pxn), B*(pxm) and D*(pxd) are defined to consist of those rows in the state matrix A, the input matrix B and the disturbance matrix D, respectively, which refer to the p decoupled states. With the above assumption we obtain,

$$A = n\overset{\longleftarrow\; n \;\longrightarrow}{\left[\begin{array}{c} A^* \\ \hline A^o \end{array}\right]}\begin{array}{l} \}\,p \\ \\ \end{array} \qquad B = n\overset{\longleftarrow\; m \;\longrightarrow}{\left[\begin{array}{c} B^* \\ \hline B^o \end{array}\right]}\begin{array}{l} \}\,p \\ \\ \end{array}$$

$$D = n\overset{\longleftarrow\; d \;\longrightarrow}{\left[\begin{array}{c} D^* \\ \hline D^o \end{array}\right]}\begin{array}{l} \}\,p \\ \\ \end{array} \tag{28}$$

The design method is based on these three, easily obtained submatrices.

Theorem:

> If B* has full rank (p) then the supremal (A,B)-invariant subspace $\mathcal{V}^*$ is equal to $\mathcal{N}$(C), the null-space of C.

If B* has full rank, then the following holds:

1) If D*=0, i.e. D* holds only null elements, then total disturbance rejection can be established through geometric feedback control $\underline{u} = F\,\underline{x}$ since $\mathcal{D} \subseteq \mathcal{V}^* = \mathcal{N}(C)$ will be satisfied.
2) If D* $\neq$ 0, then total disturbance rejection is possible through geometric feedback control combined with feedforward $\underline{u} = F\,\underline{x} + K_{FF}\,\underline{q}$. This is true, since $\mathcal{D} \subseteq \mathcal{V}^* + \mathcal{B} = \mathcal{N}(C) + \mathcal{B} = \mathcal{R}^n$, which means that eq. (16) is always satisfied, when B* has full rank.

The feedback matrix can be derived from eq. (12), which in this design method simplifies to

$$(A^* + B^*\,F)N_c = 0, \tag{29}$$

where the multiplication with N_c simply "picks out" the n-p columns of (A*+B*F) which do _not_ correspond to the decoupled states. The feedforward matrix K_{FF} can, if needed, be derived from eq. (15) which in this design method simplifies to

$$D^* + B^*\,K_{FF} = 0\,. \tag{30}$$

Accordingly, a design procedure is developed where only the submatrices A*, B* and D* are examined, followed by the solution of eq. (29) and possibly eq. (30). At the same time this procedure gives a quick answer to what is required to obtain total disturbance rejection on the system.

A STRUCTURAL APPROACH

The general principles of a graph theoretic approach to multivariable control system design have been presented by Schizas and Evans ref. (10) and related to the geometric approach. The graph theoretic, or structural, approach is by its nature restricted in scope, but it would appear to have as much to offer as the geometric analysis, and it has the added advantage of being more intuitively obvious to the designer, particularly if he is familiar with the process to be controlled.

Although useful computer aided techniques have been developed (ref. (13)), to enable the analysis of large scale systems, it is indicated here that the analysis of quite large, or complex problems can be achieved 'manually'. This analysis is not necessarily restricted to purely qualitative aspects, but it has a contribution to make to the evaluation of numerical parameters, and to establishing an economy of design. A brief introduction to the structural approach is given below. For a more comprehensive presentation the readers are referred to ref. (10).

The Use of Diagraphs in Control System Design

If we consider the set of state space equations, eq. (1), which have have been transformed, then we have

$$\underline{x}(s) = s^{-1}\underline{x}(0) + s^{-1}\,A\,\underline{x}(s) + s^{-1}\,B\underline{u}(s) \tag{31}$$

and we can interpret directly the matrices $s^{-1}A$ and $s^{-1}B$ in terms of a class of signal flow graphs which we shall term digraphs (i.e. _di_rected _graphs_).

The digraph can be drawn directly from the A and B matrices, such that the presence of any non-zero a_{ij} or b_{ij} elements indicates the existence of a directed arc of the graph from node j to node i.

Two important theorems apply which depend on the definitions of cyclic and acyclic subsystems of the graph. A cyclic subgraph is formed by a subset of the nodes and edges such that, within that subgraph, all nodes are mutually reachable along a directed path. An acyclic subgraph is one in which this is not so. The separation into these subsystems can either be done manually for simple systems or through computer programs like APL. From the graph controllability and observability can readily be determined.

The two theorems can now be stated.

Theorem 1

The eigenvalues derived from each cyclic subsystem as a digraph are eigenvalues of the complete system.

Theorem 2

All acyclic nodes are associated with structurally determined zero eigenvalues (i.e. eigenvalues which remain zero independently of the numerical values of the non-zero coefficients).

Accordingly, the transient response (i.e. the poles) depend only on the loop gains, and the existence of zeros depends on one or more of a number of possible conditions (Mason rule):

a) The number of forward paths
b) The existence of loops not touching the forward paths
c) The relative topological lengths (i.e. the number of edges) of forward paths and loops.

Hence, these last conditions determine the aspects of performance such as final steady state values. From these basic rules, ref. (10) has designed a controller for disturbance rejection for a distillation column as an example. The reader is referred to Schizas and Evans' paper for details.

The designed controller coincides with the controller previously derived in ref. (8) based on the geometric method of Wonham, but the present approach has the added advantages that useful criteria are also available concerning pole assignment, and the 'upper limits' of the design are clearly exhibited.

EVALUATION OF THE FOUR DESIGN METHODS

This section will cover the evaluation and comparison of the four previously presented design methods. All of these produce the same elements in the control matrix in order to obtain disturbance rejection. In this comparison, the designers' necessary theoretical knowledge, the condition for disturbance rejection, computer load and handling of feed forward and pole placement will be considered.

Wonham's method is founded directly on the geometrical theory. The method handles any matrices and determines if a solution exists and then always finds one. This theory will appear difficult and rather specialized with the result that few designers will master this prerequisite. The method further requires the determination of the supremal (A,B)-invariant subspace. This step will for larger technical systems be difficult and rather incomprehensible for the designer. Also, the solution of $(A+BF)\mathbf{V}^* \in \mathbf{V}^*$ is complicated. The computations can be performed on a computer, however, demanding advanced numerical methods like singular value decomposition.

Pole placement is possible, but is not integrated into the method. However, in a trial and error procedure, this can turn out to be time consuming, and if an optimal solution exists it is not guaranteed to be developed.

Shah's method is based on eigenvalues/eigenvector considerations and on some theorems developed by Shah. In this method, the designer can follow and evaluate the various steps through the design procedure. As an example, the designer can select the eigenvalues in closed loop for the decoupled states. In the case where there are more control inputs than decoupled outputs (i.e. $p<m$) the computational demand is considerable and comparable to Wonham's method. For $p=m$, the computational demand is smaller. The advantage of these methods is the handling of the pole placement since this is integrated into the design and not f.ex. a trial and error matter. If $p=m$, m eigenvalues can be arbitrarily selected, and if $p<m$, all eigenvalues can be arbitrarily selected. The method requires that B_1 has full rank, but there are restrictions in desire of freedom for pole placement. There is no direct condition for feedback alone to be sufficient for disturbance rejection. Some drawbacks are that a solution may exist even though B_1 is singular, and that determination of W_{22} from Λ_2 and W_{21} from Λ_1 for $p<m$ is a difficult mathematical operation. Finally, the selection of the submatrices, eigenvalues and eigenvectors requires insight into the problems of pole placement.

The simplified design method is founded on the geometric theory also; but it does not require that the designer possesses a general knowledge about this theory. Also, the design procedure and the computations are much less demanding than Wonham's and Shah's methods, first of all because the determination of the supremal (A,B)-invariable subspace is avoided. In the method, the linear system matrices are separated into two submatrices. The first corresponding to the selected decoupled states and the second corresponding to the rest of the states. The potential of providing complete disturbance decoupling through feedback only or if necessary supplemented with feedforward is apparent from these two submatrices. This is much more transparent than Wonham's method. Since F is determined column by column, this is fairly easy. Pole placement is possible, but is not included in the method. In comparison to Wonham's method, where the designer easily looses the insight into the system, the simplified method offers the designer greater insight into how and which elements must be determined in order to establish disturbance decoupling and which element is available for pole placement and possibly stabilization of the closed loop system. The method applies to a broader class of problems than the eigenvalue/eigenvector method, since the requirement B^* has full rank, is less restricting than the requirement B_1 is nonsingular. A drawback is that an F may exist even though B^* does not have full rank and further, there are restrictions in degrees of freedom for pole placement.

The digraph method also gives the designer good insight in and understanding of the system through which this knowledge is easily implemented. Through the digraph the designer easily notes which couplings between the different states need to be disconnected or decoupled to avoid potential disturbances. The digraph indicates directly if complete disturbance rejection is possible through feedback only or if feedforward is necessary. Drawing of the digraph is simple so it is relatively easy to tell if geometric control is sufficient or not. Design of the geometric controller is based on a number of equations, derived through the logical matrices by comparison with the element in the closed loop matrices which must be zero.

The digraph method has not integrated pole placement. For larger technical systems, however, the derivation of the many equations is difficult to overlook. Here the handling of the equations involved through matrix methods are to be preferred.

There is no simple conclusion on the previous discussion since this will depend on the weighting of the elements involved, particularly the designers' experience with and insight into the various design methods. The major arguments of the discussion is collected in the table.

CONCLUSION

Based on the evaluation above, no simple conclusion can be made, rather the choise of the most appropriate design method will depend on the background of the designer and the available computer programs. The table will here serve as an immediate available guideline for the designer. In industrial praxis the designer will consider other control methods as well. Here geometric control like many other control methods must be recognized to assume that a mathematical model is available. This will, of course, be a shortcoming in many cases. It further requires the model to possess a certain accuracy during the normal operation of the plant. Nevertheless, if these requirements are met, geometric control offers an attractive control method, particularly for disturbance rejection. The evaluation above can offer an easier introduction to the selection of the proper design method.

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REFERENCES

(1) E. Fabian and W.M. Wonham, "Decoupling and disturbance rejection", IEEE Trans. Autom. Control AC-13, 399 (1974).

(2) E. Fabian and W.M. Wonham, "Decoupling and data sensitivity", IEEE Trans.Autom.Control AC-20, 338 (1975).

(3) W.M. Wonham, "Linear multivariable control: A geometric approach", 2. ed., Springer-Verlag, New York (1979).

(4) S.L. Shah, D.E. Seborg and D.G. Fisher, "Disturbance localization in linear systems by eigenvector assignment", Int.J.Control 26, 853 (1977).

(5) S.L. Shah, D.E. Seborg and D.G. Fisher, "Design and experimental evaluation of controllers for process undisturbability", AIChE Journal 27, 131 (1981).

(6) M. Kümmel, L. Foldager and P.C. Hansen, "A simplified approach to geometric control", Preprints 9th IFAC World Congress, Budapest, Hungary, Vol. 3, 104 (1984).

(7) M. Kümmel and E. Ohrt, "Practical evaluation of geometric control", Proc. of 8th IFAC Congress, Kyoto, Japan, pp. 2679 (1981).

(8) T. Takamatsu, I. Hashimoto and Y. Nakai, "A geometric approach to multivariable control system design of a distillation column", Automatica 15, 387 (1979).

(9) M. Kümmel and L. Foldager, "Geometric control of distillation column", Preprints Applied Control and Identification, Lyngby, Denmark, pp. 3-10 (1983).

(10) C. Schizas and F.J. Evans, "A graph theoretic approach to multivariable control systems design", Automatica 17 no. 2, 371 (1981).

(11) C.L. Lawson and J.R. Hanson, "Solving least squares problems", Prentice Hall Series in Automatic Computation, Englewood Cliffs, New Jersey (1974).

(12) V.C. Klema and A.J. Laub, "The singular value decomposition: Its computation and some applications", IEEE Trans.Automat.Contr. AC-25, p.164-176 (1980).

Method	Wonham	Shah	Simplified	Digraph
Theoretical basis	The general geometrical theory as expressed by Wonham	Eigenvalue/eigenvector consideration and Shah's theorems	The geometrical theory, however, less complicated	Graph theory
Conditions for disturbance rejections	Range of D to belong to supremal (A,B)-invariant subspace	B_1 nonsingular	B* full rank	Is determined from the digraph
Designers insight through design process	Little insight immediately; high level of abstraction	Good insight since the method indicates which states to be decoupled	Like eigenvalue/eigenvector method	Like eigenvalue/eigenvector method
Computational demand	Large due to derivation of the supremal invariant subspace	For $p<m$ the demand is large. For $p=m$ the demand is moderate	Modest, since the method results in solution of simple linear equations	Like simplified
Handling of pole placement	Is not integrated	Easy. Integrated in method	Is not integrated	Is not integrated
Potential for computer solution	Well suited	Well suited	Well suited	Difficult to implement